AIAI Grant UDL Example ~ Towson University

Title: Problem Solving

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Executive Function, Universal-Design-for- Learning (UDL) Aids, for Mathematical Verbal Problems

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REFERENCE: R. Hendel*, A Discipline Independent Approach to a Higher Cognitive Pedagogy*, **Journal of Systemics, Cybernetics and Informatic***s*, 12#5, **2014** , pp 16-21.

Executive Summary

Both the Common Core State Standards and the National Council of Teachers of Mathematics (NCTM) standards emphasize the importance of verbal problems in the mathematics curriculum. Verbal problems span the curriculum from K-12 through both the undergraduate and graduate experiences. Yet many students cannot deal with verbal problems even though their symbolic and verbal skills are separately adequate. This vignette contains a Universal Design for Learning (UDL) aid. The aid focuses on strengthening executive function, a necessary cognitive requirement for solving verbal problems which definitionally involves two brain areas, the verbal and mathematical. Significant success has been achieved with using this aid; students who formerly had severe anxiety in dealing with verbal problems were now successfully able to solve them. The aid seeks to train students to focus on keywords in the verbal problem and then translate it to mathematics using a verbal-mathematics dictionary. The technique works best on students with minimum verbal and algebraic competency who have difficulty solving verbal problems. The illustrative example illustrates this technique.

Illustrative Example:

We use the following verbal problem which could easily occur in the K-12 curriculum.

> John paid $7 for two Cokes and three bags of potato chips. Sharon, at the same store, also paid $7 but purchased three Cokes and one bag of potato chips. How much does a Coke cost? How much does a bag of potato chips cost?

Illustrative Solution:
Although the *ultimate* goal is to solve the problem, an important intermediate goal is to *translate* the problem into pure algebra. Towards this end, we need a *translation prompt*, if you want a dictionary. The dictionary contains keywords that frequently occur in verbal problems that prompt the experienced student to *translate* to algebra. We use a three-step process consisting of

1) Rereading the problem and underlining keywords
2) Use of a mathematical dictionary to translate these keywords
3) Translation of the *verbal* problem into *algebra.*

Step 1) We restate the problem below with keywords underlined and provide the dictionary afterwards.

> John paid $7 for two Cokes and three bags of potato chips. Sharon, at the same store, also paid $7 but purchased three Cokes and one bag of potato chips. How much does a Coke cost? How much does a bag of potato chips cost?

This process is not meant to be mechanical but rather is in the nature of a prompt. As can be seen, the following words or word phrases are underlined: i) all numbers, ii) phrases beginning with a number, iii) keywords like `and, or, greater than, less than` iv) keywords like `for, is` etc.

Step 2) The corresponding dictionary might have entries are follows:

Keyword	**Numbers**	**Phrases with numbers**	**Keyword: `and`**	**Keywords like `for, is`**
Algebraic translation	Numbers	Number x Variable representing the objects (nouns) in the phrase	Arithmetic summation (+)	Equality operator (=)
Example:	1,2,7,3 become 1,2,7,3	2 Cokes=2C 1 Coke = C 3 Bags = 3B 1 Bag = B	2 cokes `and` three bags of potato chips becomes 2C+B	Paid 7 `for` 2 cokes and three bags of potato chips; 7=2C+B
Comments		Variable meaning must be identified. Here *C, B* refer to the number of Cokes and bags of potato chips bought	Interestingly, the word `and` translates smoothly into +; however, the phrase less than does not translate smoothly into minus	The process is not mechanical: Not all `for, is` can be translated as equality. Understanding of context must be used.

Step 3) We now translate the entire problem. The table below emphasizes how the previous steps assist us in translation.

Verbal problem	**Corresponding Algebraic Statement**
John paid $7 for two Cokes and three bags of potato chips	7 = 2C+3B

Sharon, at the same store, also paid $7 but purchased three Cokes and one bag of potato chips.	7 = 3C+B
How much does a Coke cost? How much does a bag of potato chips cost?	Solve for C, the variable representing the number of cokes bought and B, the variable representing the number of bags bought

General Discussion: There are additional points to be made on the above process.

- Prompt vs. Mechanical: How is this process to be used? It is not meant to be mechanical. Rather it is presented as a prompt. A student practicing this approach on many verbal problems will expand the horizons of their executive function and can transfer the skills learned to new situations
- Supporting evidence:(The author has strong anecdotal and qualitative experience supporting this method; no thorough quantitative testing of this method however has been done. The method has enabled students who repeatedly failed a math course to pass with satisfactory grades).
- Applicability: The method is applicable to a wide variety of verbal problems spanning the K-12 curriculum and college verbal problems such as those found in introductory statistics and probability courses.
- Non-smooth translations: As already pointed out, not all translation is smooth. For example, *2 cokes and 3 bags of potato chips* translates smoothly into 2C+3B; contrastively, *an unknown number is 3 less than 10* does not translate into U=3-10 but rather translates into U= 10-3. Here the relational phrase *less than* has the effect of reversing the order of the English phrases it relates.
- Executive Function: Note that we have *not* completed solving the problem! We have performed an important first step, the *translation* of the verbal problem to algebra. The key to this translation is emphasis on executive function skills. The author has found it helpful to explain to students who have difficulty with word problems that *The reason you can read well and solve algebra well is because they involve one area of the mind; contrastively, a verbal problem intrinsically involves two areas of the brain, the verbal and formal. The performance of any activity simultaneously involving several brain areas requires executive function and is considered a higher form of intellectual challenge.*
- Verbal-Algebra: Finally, we emphasize that *after* translating the verbal problem into an algebraic problem, one must solve the algebraic problem.
    - 7 = 2C+3B
    - 7 = 3C + B

  The methods for solving such a system are distinct from the methods and skills needed to translate the verbal problem. Thus, we regard the solution as a secondary aspect of the problem and have identified the above technique as a prompt to assist in the overall problem solutions.