# The Experimental Verdict on Spacetime from Gravity Probe B

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**Abstract** Concepts of space and time have been closely connected with matter since the time of the ancient Greeks. The history of these ideas is briefly reviewed, focusing on the debate between "absolute" and "relational" views of space and time and their influence on Einstein's theory of general relativity, as formulated in the language of four-dimensional spacetime by Minkowski in 1908. After a brief detour through Minkowski's modern-day legacy in higher dimensions, an overview is given of the current experimental status of general relativity. Gravity Probe B is the first test of this theory to focus on spin, and the first to produce direct and unambiguous detections of the geodetic effect (warped spacetime tugs on a spinning gyroscope) and the frame-dragging effect (the spinning earth pulls spacetime around with it). These effects have important implications for astrophysics, cosmology and the origin of inertia. Philosophically, they might also be viewed as tests of the propositions that spacetime acts on matter (geodetic effect) and that matter acts back on spacetime (frame-dragging effect).

## 1 Space and Time Before Minkowski

The Stoic philosopher Zeno of Elea, author of Zeno's paradoxes (c. 490-430 BCE), is said to have held that space and time were unreal since they could neither act nor be acted upon by matter [1]. This is perhaps the earliest version of the *relational view* of space and time, a view whose philosophical fortunes have waxed and waned with the centuries, but which has exercised enormous influence on physics. The opposing *absolutist view*, that space and time do possess independent existence apart from matter, has an equally distinguished history that might be traced back to the Stoics' philosophical rivals, the Epicureans, whose founder Leucippus of Abdera (active c. 450 BCE) introduced the concept of a pre-existing void as the "emptiness between atoms" [2]. The earliest explicit statement of the absolutist view has been attributed by Max Jammer to the Pythagorean philosopher Archytas (428-347 BCE): "Since everything which is moved into a certain place, it is plain that the place where the thing moving or being moved shall be, must exist first" [3].

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Aristotle (384-322 BCE) constructed a hybrid of the absolute and relational views. He accepted arguments similar to that of Archytas, but was deeply unhappy with the atomistic idea of void, “since no preference can be given to one line of motion more than to another, inasmuch as the void, as such, is incapable of differentiation … how [then] can there be any natural movement in the undifferentiated limitless void?” To get around this difficulty Aristotle developed the arguably relational idea that space is defined by that which contains it. He was led in this way (in the *Physics*) to his influential picture of a cosmos pinned simultaneously to the center of the earth and the firmament of fixed stars: “The center of the universe and the inner surface of the revolving heavens constitute the supreme ‘below’ and the supreme ‘above’; the former being absolutely stable, and the latter constant in its position as a whole.” Such was Aristotle’s authority that few questioned it for two millenia. An exception was John Philoponus (c. 490-570), who argued for a more purely absolute picture and reacted in particular against the idea that space is somehow defined by that which contains it: “Place is *not* the adjacent part of the surrounding body … It is a given interval, measurable in three dimensions; it is distinct from the bodies in it, and is, by its very nature, incorporeal. In other words, it is the dimensions alone, devoid of any body.”

Claudius Ptolemy (c. 85-165) elaborated on Aristotle’s system, using only circular motions and uniform speeds so as to “save the phenomena” in the face of increasingly accurate observations. However, the way in which he did so points up the limited extent to which Aristotle’s thinking can truly be considered relational. The fact that the “firmanent of fixed stars” and “center of the earth” defined the rest frame of Aristotle’s cosmos did not mean that space was *physically* anchored to the matter making up the earth or stars. Rather it so happened that these referents stood still in a background space that was more properly conceived as existing absolutely. Thus, adopting an earlier idea of Hipparchus, Ptolemy first detached the sun’s “orbit” from the center of the earth (giving it an “eccentricity”). Later he added planetary “deferents,” “epicycles” and finally “equants”–all reference points or paths in *empty space* (some of them even with inherent motions of their own). These so-called “void points” make sense only with respect to absolute space–or perhaps to “matter” of a divine kind, as hinted at in the *Almagest*: “The first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity.” Here Ptolemy anticipated Newton, who would later refer to absolute space (in the *Opticks*) as the “sensorium” of God.

The nature of time as well as space was eagerly debated in this way by the ancients. The Epicurean philosopher Lucretius (c. 99-55 BCE) may have been the first to argue explicitly for a relational view of time, writing in *The Nature of the Universe* that: “Time by itself does not exist … It must not be claimed that anyone can sense time by itself apart from the movement of things.” Saint Augustine (354–430) put a theological twist on this argument in his *Confessions*, emphasizing that “God created the world *with* time, not in time.”

Nicolaus Copernicus (1473–1543) relocated the center of Aristotle’s universe from the earth to the sun. This step was not quite so daring as often thought, for Hipparchus and Ptolemy had already nudged the sun’s “orbit” away from the center of the earth by introducing “eccentricity.” As Copernicus himself noted near the beginning of *De Revolutionibus*: "Nothing prevents the earth from moving ... For, it is not the center of all the revolutions." Furthermore, although he re-centered the cosmos kinematically on the sun, Copernicus did not attach space dynamically to the rest frame of the sun or any other physical body, but followed Aristotle in associating it with the metaphysical "sphere of the fixed stars," which (he wrote): "contains itself and everything, and is therefore immovable. It is unquestionably the place of the universe, to which the motion and position of all the other heavenly bodies are compared."

Fifty years later, the notion of rigid planetary spheres could no longer be reconciled with astronomical observations, leading Johannes Kepler (1571–1630) to declare: "From henceforth the planets follow their paths through the ether like the birds in the air. We must therefore philosophize about these things differently." Thoughts such as these led him to the radical idea of attaching the rest frame of space to *physical bodies* rather than a metaphysical construct such as absolute space (he conceived of forces extending outward from the sun and sweeping the planets along in their orbits). The laws of planetary motion that he subsequently derived have been wonderfully characterized by Julian Barbour as a "pre-Machian triumph of Mach's Principle" [2].

A similar shift in thinking is apparent in Galilei Galileo (1564–1642). Rather than identifying the fixed stars with the rest frame of space in an abstract sense, he asserted (in the *Dialogo*) that they are *physically* at rest in space: "The fixed stars (which are so many suns) agree with our sun in enjoying perpetual rest." However, Galileo did not further define this state of "rest," and appears to have implicitly adopted the absolutist view. In fact he was the first to use the actual term "absolute motion," in his theory of the tides. Réné Descartes (1596–1650) also relied on the concept of absolute space (which he referred to as a "plenum") in arriving at something similar to Newton's eventual first law of motion. After learning of Galileo's trial by the Inquisition, however, he put off publishing his results by more than a decade and eventually prefaced them (in the *Principia Philosophiae*) by a disclaimer stating that all motion was, after all, relative. He may have been the first to hold both absolutist and relational views at the same time.

This inconsistency irritated Isaac Newton (Fig. 1), who complained in *De Gravitatione* that if all motion was really relative as Descartes said, then "it follows that a moving body has no determinate velocity and no definite line in which it moves." It was partly to do away with any such confusion that he expressed himself so

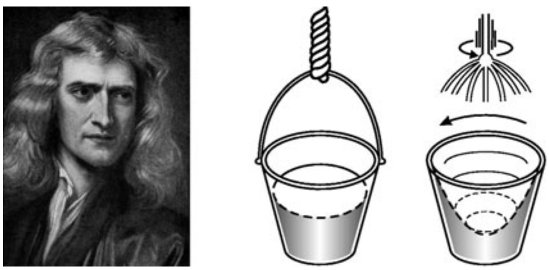

**Fig. 1** Isaac Newton (1643–1727) and his bucket experiment: the concavity of the water's surface indicates that the water is rotating with respect to "absolute space"

categorically in the famous opening of his *Principia*: “Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external … absolute space, in its own nature, without relation to anything external, remains always similar and immoveable.” He added that the existence of absolute space could be demonstrated by watching the water in a spinning bucket. The fact that the water’s surface gradually assumed a concave shape showed that it was spinning with respect to *something*; how else would it know what to do? Proof of the reality of space, in other words, could be found in the inertia of matter.

Newton’s most formidable relational critic was the mathematician and philosopher Gottfried Wilhelm Leibniz (1646–1716), who retorted (in a letter to Christiaan Huygens): “If there are 1,000 bodies, I still hold that … each separately could be considered as being at rest … Mr. Newton recognizes the equivalence of hypotheses in the case of rectilinear motion, but with regard to circular motion he believes that the effort which revolving bodies make to recede from the axis of rotation enables one to know their absolute motion. But I have reasons for believing that *nothing* breaks this general law of equivalence” [4]. The philosopher Bishop George Berkeley (1685–1753) went even farther, writing in *De Motu* that the very concept of two bodies “moving” around a common center is meaningless in empty space, since a co-rotating observer will not see anything change. “Suppose,” however, “that the sky of fixed stars is created; suddenly from the conception of the approach of the globes to different parts of the sky the motion will be conceived.”

If Newton’s was the definitive statement of the absolutist view of space, then his most notorious relational counterpart was Ernst Mach (Fig. 2), who addressed himself directly to Newton’s bucket argument, writing in *The Science of Mechanics*: “No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass until they were ultimately several leagues thick.” A sufficiently large or massive bucket, in other words, might carry the local inertial frame of the water around *with it* and leave the water’s surface flat. This was perhaps the first explicit, though physically incomplete suggestion of a phenomenon now generally referred to as *frame-dragging*.

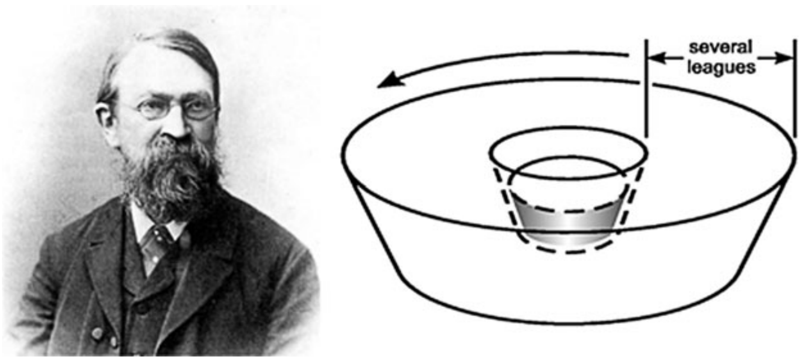


**Fig. 2** Ernst Mach (1838–1916) and his revision of Newton’s bucket experiment: would the water still climb up the walls if the bucket were *arbitrarily large and massive?*

Mach's principle, as his rather vague suggestion has come to be known, has proved stubbornly difficult to formulate in a precise physical way, and even more difficult to test experimentally. At a conference on this subject in Tübingen in 1993, leading experts discussed at least 21 different versions of "Mach's principle" in the scientific literature, some of them mutually contradictory [5]. It is probably for this reason that Mach's relational ideas have proved to be more inspirational than fruitful in physics. Nevertheless they led to some fascinating experimental investigations, even before Einstein's time. In 1894 the German vulcanologist Immanuel Friedländer (1871–1948) and his brother Benedict (1866–1908) looked for evidence that heavy rotating millstones could exert a Mach-type force on a sensitive torsion balance, and confessed (in *Absolute or Relative Motion?*) that they could find no definite results either way. In 1904, fellow German physicist August Föppl (1854–1924) published the results of an experiment designed to detect the influence of the rotating Earth on the angular momentum of a pair of heavy flywheels whose spin axis could be aligned along either lines of latitude or longitude (Fig. 3). He too found nothing, but noted that his accuracy was limited to about 2%.

Experiments like Gravity Probe B should not be seen as tests of Mach's principle (which is ill-defined as it stands), but rather as tests of specific theories of gravity (which may or may not incorporate well-defined "Machian" features such as frame-dragging). Nevertheless, it is possible to think of Gravity Probe B as a realization of the experiment suggested by Mach (and actually attempted by Föppl) in which the role of the "bucket" is played by the earth and the dragging of local inertial

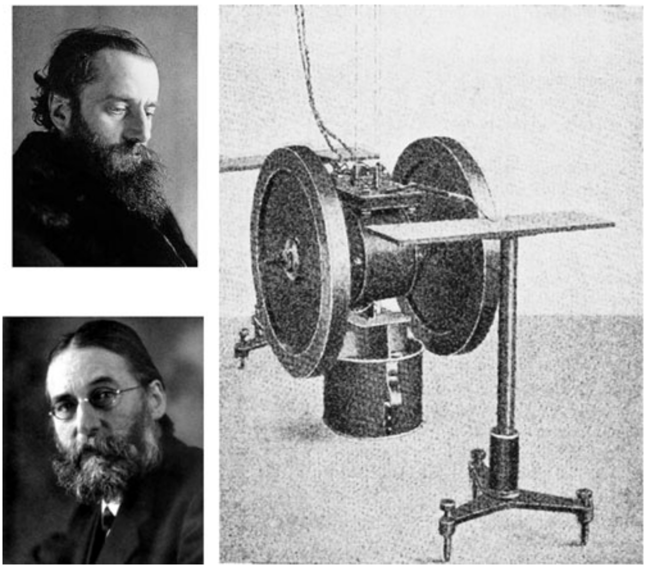

**Fig. 3** Early experimenters in frame-dragging: Benedict Friedländer (*top left*), August Föppl (*bottom left*), Föppl's experimental apparatus (*right*)

frames is measured not by water but by orbiting gyroscopes over a million times more sensitive than the best navigational gyros on earth.

## 2 Spacetime After Minkowski

Albert Einstein (1879–1955) radically re-ordered the traditional priorities of metaphysics when he showed in 1905 that there is a quantity more fundamental than either space or time, namely the speed of light $c$. Space and time are interconvertible, and must be so in order to preserve the constancy of $c$ for all observers. The geometrical inference that space and time could be seen as components of a single four-dimensional spacetime fabric came from Hermann Minkowski (Fig. 4), who announced it in Cologne 100 years ago with the words:

> "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

Einstein initially dismissed Minkowski's four-dimensional interpretation of his theory as "superfluous learnedness" [6]. To his credit, he quickly changed his mind. The language of spacetime (tensor calculus) proved to be essential in making the transition from special to general relativity.

This transition required two main steps, a physical one and a mathematical one, and both relied crucially on Minkowski's spacetime picture. The physical step occurred in 1907 when, in the patent office in Bern, Einstein was struck by what he later called his "happiest thought": a man falling off the side of a building feels no gravity. The significance of this observation lies in the fact that the *same* choice of accelerated coordinates suffices to transform away the earth's gravitational field, *regardless of who or what is dropped*. If gravity were like any other force–electromagnetism, say–differently charged objects would "fall" quite differently, some of them even accelerating upward. By contrast, gravity appears matter-blind. From this observational fact (now known as the *equivalence principle*) Einstein leapt to the spectacular inference that gravitation must originate, not in any property of matter, but in spacetime itself. He eventually identified the relevant property of

**Fig. 4** Hermann Minkowski (1864–1909)

spacetime as its curvature. This idea is the physical foundation of general relativity, succinctly summarized by John Wheeler as: "Spacetime tells matter how to move; matter tells spacetime how to curve." But while the resulting theory has been very successful, Einstein himself saw it as incomplete. In particular, he was unhappy with its dualistic division of physical reality into "spacetime" and "matter," describing these in 1936 as being like two wings of the same building, one made of "fine marble ... the other of low-grade wood." In the 1956 edition of *The Meaning of Relativity*, published in the year after his death, he still expressed the belief that this distinction would prove to be a temporary one: "In reality space will probably be of a uniform character and the present theory be valid only as a limiting case." If indeed matter and spacetime could be described as aspects of a single, unified field (as many physicists still hope), the very philosophical distinction between "relational" and "absolute" points of view might lose its meaning.

The second, more mathematical step toward general relativity was the search for a way to describe the dynamics of curved spacetime in a way that would hold for all observers–even accelerating ones–regardless of their choice of coordinates. By contrast with the equivalence principle, this principle (known as *general covariance*) did not arrive in a flash but required years of difficult slogging through the forest of tensor analysis (Fig. 5). Einstein memorably described the goal of expressing physical laws without coordinates as "equivalent to describing thoughts without words."

Today it is commonplace to speak of equivalence and general covariance as the two foundations of general relativity. In 1918, however, Einstein himself identified a third, philosophical pillar of his theory: *Mach's principle*. This characterization is now widely regarded as wishful thinking. Einstein was undoubtedly inspired by Mach's relational views, and initially hoped that his new theory of gravitation would "secure the relativization of inertia" by binding spacetime so tightly to matter that one could not exist without the other. In fact, however, the equations of general relativity are perfectly consistent with spacetimes that contain no matter at all. Flat (Minkowski) spacetime is a trivial example, but empty spacetime can also be curved, as demonstrated by Willem de Sitter in 1916. There are even spacetimes whose distant reaches rotate endlessly around the sky relative to an observer's local inertial frame, as demonstrated by Kurt Gödel in 1949. The bare existence of such solutions

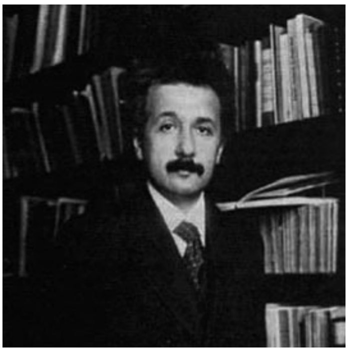

**Fig. 5** Albert Einstein in 1916

in Einstein's theory shows that it cannot be Machian in any strong sense; matter and spacetime remain logically independent. The term "general relativity" is thus something of a misnomer, as emphasized by Minkowski and others since. The theory does *not* make spacetime more relational than it was in special relativity. Just the opposite is true: the absolute space and time of Newton are retained. They are merely amalgamated and endowed with a more flexible mathematical skeleton (the metric tensor). When this became clear, Einstein's interest in Mach faded, and he wrote to a colleague in 1954: "As a matter of fact, one should no longer speak of Mach's principle at all."

Nevertheless, Einstein's theory of gravity represents a major swing back toward the relational view of space and time, in that it answers the objection of the ancient Stoics. Space and time *do* act on matter, by guiding the way it moves. And matter *does* act back on spacetime, by warping and twisting it. Perhaps nowhere is this more strikingly illustrated than in the two effects Gravity Probe B is designed to detect directly for the first time: the *geodetic effect*, in which curved spacetime around the massive earth causes an orbiting gyroscope to precess about an axis perpendicular to the plane of its motion; and the *frame-dragging effect*, in which the rotating earth pulls spacetime around with it, twisting the gyroscope's spin axis along the equatorial plane (Fig. 6). In that sense, general relativity is indeed nearly as relational as Mach might have wished. Some physicists, most notably Julian Barbour, have asserted that general relativity is in fact perfectly Machian, at least for closed (i.e. finite) spacetimes [7]. Key to this claim is the argument that allegedly "un-Machian" empty spacetimes (like those of Minkowski and de Sitter) are idealizations that do not take gravitational degrees of freedom into account. (The idea that gravitational radiation is responsible for transmitting inertia between mutually accelerated masses has been explored by Dennis Sciama [8], John Wheeler [9] and

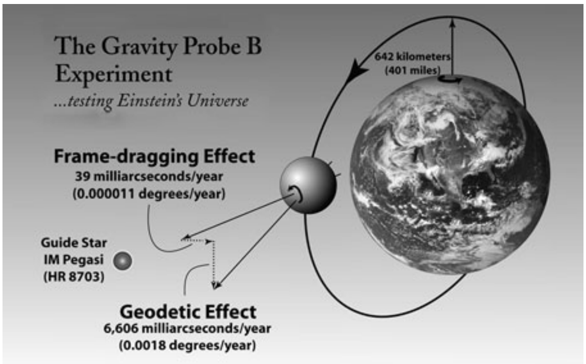


**Fig. 6** In Einstein's theory of general relativity, spacetime acts on matter through its curvature, causing the spin axis of a gyroscope in orbit around a large mass like the earth to "fall into" the direction of travel (geodetic effect). Matter acts back on spacetime, not only by curving it, but also by *pulling spacetime with it*, causing the spin axis of a gyroscope to precess in the direction of the earth's rotation (frame-dragging effect). Gravity Probe B is designed to detect both of these effects directly and unambiguously for the first time

others [10].) In the context of modern quantum field theory, the distinction between absolute and relational views of spacetime breaks down as "empty space" becomes populated not only by gravitational waves but also by matter in the form of virtual particles, zero-point fields, etc. [11]. Within the classical world of Minkowski and Einstein, however, the majority view might best be summed up as follows: *spacetime behaves relationally but exists absolutely.*

## 3 Minkowski's Legacy in Higher Dimensions

By uniting space and time in a common metrical framework Minkowski shattered the prejudice, going back to the ancient Pythagoreans, that geometry applies only to *lengthlike quantities*. He was the first to make such a proposal in the context of a fully realized physical theory (special relativity) and it is entirely appropriate to consider him the father of spacetime. Nevertheless there were intriguing precursors for such a union before 1908, and these may have helped to prepare the conceptual ground for the eventual acceptance of relativity theory.

Possibly the first to refer to time as a fourth dimension was the French mathematician Jean d'Alembert, in an article in the *Encyclopédie* that he co-edited with Denis Diderot in 1754. Mysteriously, d'Alembert attributed the idea to "an enlightened man of my acquaintance" [12]. This unnamed source is thought to be the French-Italian mathematician Joseph-Louis Lagrange, who though only 18 years old at the time of the publication of the *Encyclopédie*, later observed in *Theory of Analytical Functions* (1797) that with time as a fourth coordinate "one can regard mechanics as four-dimensional geometry."

The German philosopher Arthur Schopenhauer referred repeatedly to matter, motion and causation as equivalent to the "union of space and time" in *The World as Will and Representation* (1818). He was, however, not concerned with physics, but rather with staking out a philosophical position relative to his predecessor Immanuel Kant. By equating these concepts Schopenhauer aimed to reduce the number of mental categories that Kant had argued were necessary for the mind to make sense of experience. For both thinkers space and time were "united" mainly in the sense that they existed more as forms of perception than as features of any external reality. Schopenhauer in turn exerted tremendous influence on the composer Richard Wagner, whose opera *Parsifal* (1877) contains this fascinating exchange between two knights on their way to the temple of the holy grail: "I barely tread, yet seem already to have come so far ... You see, my son, time here becomes space."[1]

[1] Much scholarly ink has been spilt on this passage by Wagner; see for instance Hans Melderis' *Space-Time-Myth: Richard Wagner and modern science* [14]. The composer's debt to Kant and Schopenhauer is suggested by a letter he wrote while working on *Parsifal* in 1860: "Since time and space are merely our way of perceiving things, but otherwise have no reality, even the greatest tragic pain must be explicable to those who are truly clear-sighted as no more than the error of the individual" [13].

Ideas of unifying space and time were not restricted to Europe, as evidenced by this line from the prose poem *Eureka* (1848) by the American author Edgar Allan Poe: "… the considerations through which, in this Essay, we have proceeded step by step, enable us clearly and immediately to perceive that Space and Duration are one." This has sometimes been interpreted as a prophetic anticipation of relativity theory,[2] but it is likely that Poe was merely stressing in a literary way that the size and age of the visible universe are correlated via the speed of light (a fact that he used elsewhere in *Eureka* to present the germ of the first scientifically correct solution to Olbers' paradox in astronomy [16]).

A more mathematical precursor to the spacetime concept is found in the generalizations of complex numbers known as quaternions, invented by the Irish physicist and mathematician William Rowan Hamilton in 1843. The fact that these objects consist of one real (scalar) component plus an imaginary (three-vector) component led Hamilton to argue as follows: "Time is said to have only one dimension, and space to have three dimensions … The mathematical quaternion partakes of both these elements; in technical language it may be said to be 'time plus space,' or 'space plus time' " [17]. But probably the most explicit anticipation of Minkowski came from "S.," an anonymous contributor to the British journal *Nature* in 1885, who wrote: "… there is a new three-dimensional space for each successive instant of time; and, by picturing to ourselves the aggregate formed by the successive positions in time-space of a given solid during a given time, we shall get the idea of a four-dimensional solid …" [18]. "S." was likely the English mathematician James Joseph Sylvester [19]. In the wake of articles such as this, the idea of time as a fourth dimension seeped into public awareness, culminating in novels like H.G. Wells' *The Time Machine* (1895), whose hero opens the book by telling his listeners that "there is no difference between Time and any of the three dimensions of Space except that our consciousness moves along it."[3] One final illustration of the extent to which spacetime was in the air prior to Minkowski's pronouncement is the "New theory of space and time" (1901) of Hungarian philosopher Menyhért Palágyi, in which space and time were combined in a four-dimensional "flowing space" by means of mixed coordinates $x + it, y + it, z + it$ [20].[4]

Minkowski's 1908 geometrization of time via the relation $x^0 = ct$ was, of course, physically motivated by Einstein's successful union of Newtonian mechanics and Maxwellian electromagnetism in the form of special relativity. Given our present mania for further kinds of unification in higher dimensions, it is surprising that more physicists have not taken Minkowski's example to heart and attempted to expand the domain of geometry *beyond space and time.*

[2] Einstein was apparently familiar with Poe's *Eureka*, referring to it in 1934 as "a beautiful achievement of an unusually independent mind" [15].

[3] Hubert Goenner [20] makes the interesting observation that Minkowski could have read Wells' *Time Machine*, as it appeared in German translation in 1904.

[4] After learning of Minkowski's speech in 1908, Palágyi attempted unsuccessfully to claim priority for the discovery of spacetime.

Historically, the reluctance to consider new kinds of coordinates is a practical one: we see no evidence for extra dimensions at experimentally accessible scales of length, time and energy. The same objection applied in Minkowski's day. The reason why time and space appeared independent until 1908 is that the size of the dimension-transposing constant "$c$" that converts one into the other is many orders of magnitude larger than the characteristic speeds of everyday life. The main effect of the new coordinate in four-dimensional (4D) special relativity is to multiply familiar (non-relativistic) quantities by the factor

$$\gamma_{4D} = \frac{1}{\sqrt{1 - (dx/c\,dt)^2}}. \tag{1}$$

When $dx/dt \ll c$, as is true nearly everywhere on earth outside modern particle accelerators, then $\gamma_{4D} \approx 1$ and spacetime looks like space.

Inspired by the unification of mechanics and electromagnetism in four dimensions, the Finnish physicist Gunnar Nordström (1914) and the German mathematician Theodor Kaluza (1921) hit upon the idea of further unifying electromagnetism and *gravity* by means of a fifth lengthlike coordinate $x^5 = \ell$ (Fig. 7).[5] Nordström's was a scalar theory of gravity that was soon proven incompatible with observation. Kaluza's, however, was a five-dimensional (5D) extension of Einstein's tensor theory. The resulting theory turned out to contain both standard general relativity *and* Maxwell's electromagnetism in four dimensions, a miracle that is nowadays understood as arising from the fact that U(1) gauge invariance is "added onto" Einstein's theory in the guise of invariance with respect to coordinate transformations along the extra dimension. To explain why this new coordinate is not seen in nature, Kaluza imposed a "cylinder condition" whereby 4D physics is essentially independent of $\ell$ by fiat. The Swedish physicist Oskar Klein (1926) showed that this independence could arise in a more natural way if the new coordinate had a circular topology

**Fig. 7** First to consider extending Minkowski's spacetime with a fifth dimension: Gunnar Nordström (*left*), Theodor Kaluza (*center*) and Oskar Klein (*right*)

[5] The superscript "4" is generally reserved for an imaginary version of Minkowski's fourth coordinate $x^0$, written as $x^4 = ict$, which allows the metric of flat Minkowski space to be written in Euclidean form.

and a compact scale (below $\sim 10^{-18}$ cm). Compactified Kaluza-Klein theory was picked up by Einstein and Bergmann (1938), Jordan (1947) and others, and eventually re-emerged as the basis for nearly all higher-dimensional unified theories today, including string and M-theory [21].

However, while it has been immensely influential, Nordström, Kaluza and Klein's idea is less radical than Minkowski's in that the proposed new coordinate shares the lengthlike character of ordinary three-space. Philosophically, this represents a return to the Pythagorean prejudice that geometry should deal only in quantities that can be measured with a meter-stick. Others have been bolder. The remainder of this section is intended as a brief and undoubtedly incomplete introduction to some of the *non*-lengthlike coordinates that have been considered in the literature. Among the earliest such proposals were those of W. Band (1939) and O. Hara (1959) relating $x^5$ to particle *spin* [22, 23], and that of Y.B. Rumer, who proposed in 1949 a fifth coordinate based on *action* via $x^5 = S/mc$ [24]. Rumer applied this idea to what he termed "5-optics" and imposed a restriction (called the "requirement of physical admissibility") similar to Kaluza's cylinder condition [25]. Related work has been done more recently by Yu and Andreev [26].

At its most fundamental, physics deals with dimensions of length [L], time [T] and mass [M], so the most natural choice for a new "post-Minkowskian" coordinate is arguably one related to *mass* via either $x^5 = Gm/c^2$ or $x^5 = h/mc$. Newton's gravitational constant $G$ (or alternatively Planck's constant $h$) is thereby promoted to the same dimension-transposing role as "$c$" in 4D special relativity. This proposal is most closely associated with P.S. Wesson and his collaborators beginning in 1983 [27, 28], though related ideas were discussed as early as 1967 by de Vos and Hilgevoord [29] and 1974 by Edmonds [30, 31].[6] In the context of *non*-compactified or Space-Time-Matter (STM) theory, where Kaluza's cylinder condition is relaxed in principle, the identification of $x^5$ with rest mass is suggested by several lines of argument including the fact that the 4D relativistic energy-momentum relation $p^\alpha p_\alpha = E^2 - c^2 p^2 = m^2 c^4$ reduces simply to $p^A p_A = 0$ in 5D (where $\alpha = 0, 1, 2, 3$ and $A = 0, 1, 2, 3, 5$); and that the 4D free-particle action principle $\delta(\int m ds) = 0$ is contained in the simpler 5D one $\delta(\int dS) = 0$ (where $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ and $dS^2 = g_{AB} dx^A dx^B$) [21, 33, 34]. The size of the dimension-transposing constant $G/c^2$ provides a natural explanation for the fact that no mass-like fifth dimension has yet been detected. "Velocity" along such a direction means *varying rest mass*, and the 5D generalization of (1) reads:

$$\gamma_{5D} = \frac{1}{\sqrt{1 - (1/c^2)(dx/dt)^2 \pm (G^2/c^6)(dm/dt)^2}}. \tag{2}$$

The factor $1/c^2$ is already small enough that time was not recognized as part of space until 1908. The factor $G^2/c^6$ is so much smaller yet (by 54 orders of magni-

[6] The idea that $x^5$ might be related to mass has also been independently attributed to M.A. Neacsu in 1981 [32].

tude in SI units) that it is no surprise that possible variation in, say, the rest masses of elementary particles has yet to be observed. Such variation, if it exists, likely takes place on cosmological scales. STM theory is consistent with the classical tests of general relativity in the solar system as well as cosmological and other experimental data [35–37]. The status of mass as a fifth coordinate has been most recently reviewed by Wesson in 2003 [38].

Additional *time*like dimensions in the context of Kaluza-Klein theory lead to the wrong sign for the Maxwell action, and to the appearance of tachyons (negative-mass eigenstates) in the theory. More generally, extra temporal dimensions raise the specter of causality violation via closed timelike curves (CTCs). For these reasons few physicists have taken Minkowski's example literally enough to posit additional coordinates with the dimensions of time (i.e., $x^5 = c\tau$). The earliest such proposal may be that of A.D. Sakharov in 1984 [39]. Sakharov considered even numbers of additional compact time dimensions and argued that causality could be preserved for macroscopic processes if the radius of compactification were suitably small. This work was further developed by Aref'eva and Volovich [40]. Other "two-time" theories have been propounded by Burakovsky and Horwitz [41], Bars and Kounnas [42], Wesson [43], Kociński and Wierzbicki [44], Erdem and Ün [45] and Quiros [46].

Other ways of extending Minkowski's four-dimensional spacetime have been considered as well. Fukui [47] studied the possibility of extra coordinates proportional to both mass and *charge* via $x^5 = \sqrt{G/c^4}\,q$. Such an identification goes somewhat against the spirit of Kaluza's original theory, in which electric charge arises in the form of momentum along $x^5 = \ell$. M. Carmeli, in his theory of cosmological special relativity [48], proposed a fifth coordinate proportional to *cosmological recession velocity* via $x^5 = v/H$, where $H$ =Hubble's constant (the expansion rate of the universe). The resulting theory is intended to supplant general relativity on cosmological scales where, for example, it has been claimed to predict cosmic acceleration [49] and obviate the need for dark matter [50]. Additional kinds of "post-Minkowskian" coordinates have been explored by Redington [32], Matute [51] and Delbourgo [52] and others.

It is too early to say whether any of these candidate coordinates will eventually prove to be as useful as Minkowski's in unifying the laws of physics. If and when that happens, we may find ourselves amending his speech of 100 years ago only slightly so that, for instance: "Space and time by themselves, *and mass by itself*, are doomed to fade away into mere shadows, and ony a kind of union of the *three* will preserve an independent reality."

## 4 Experimental Tests: An Unfinished Job

General relativity, based on a flexible and animated version of Minkowski's four-dimensional spacetime, has survived over 90 years of experimental test. Nevertheless there are at least four good reasons to think that the theory is incomplete and must be overthrown just as Newton's was. First, general relativity predicts its own demise; it breaks down in *singularities*, regions where the curvature of spacetime becomes infinite and the field equations can no longer be applied. These cannot be dismissed as mere academic curiosities, because they do apparently occur in the real universe if general relativity holds. Theoretical work by S. Hawking, R. Penrose and others has proven that singularities must form within a finite time (the universe is necessarily "geodesically incomplete"), given only very generic assumptions such as the positivity of energy. Two places where we expect to find them are at the big bang, and inside black holes like the one at the center of the Milky Way. If we are to fully understand these phenomena, then general relativity must be modified or extended in some way.

Second, there is the question of cosmology. Under the reasonable assumptions that the universe on large scales is homogeneous and isotropic (the same in all places and in all directions), as implied by observation in combination with the Copernican principle, general relativity has led to a cosmological theory known as the big bang theory. This theory has had some spectacular successes; for instance, the prediction of the cosmic microwave background radiation, the calculation of the abundances of light elements, and a basis for understanding the origin of structure in the universe. It also has some weaknesses, notably involving finely tuned initial conditions (the "flatness" and "horizon problems"). More troublingly, in recent decades it has become impossible to match the predictions of big-bang cosmology with observation unless the thin density of matter observed in the universe (i.e. that which can be seen by emission or absorption of light, or inferred from consistency with light-element synthesis) is supplemented by much larger amounts of unseen *dark matter* and *dark energy* that cannot consist of anything in the standard model of particle physics. The observations are quite clear: the required exotic dark matter has a density some five times that of standard-model matter, and the required dark energy has an energy density some three times greater still. To date, there is no direct experimental evidence for the existence of either component, and there are strong theoretical reasons (the "cosmological constant problem") to be suspicious of dark energy in particular. There is also no convincing explanation of why two new and as-yet unobserved forms of matter-energy should be so closely matched in energy density (the "coincidence problem"). While the majority of cosmologists seem prepared to accept both dark matter and dark energy as necessary, if inelegant facts of life, others are beginning to interpret them as possible evidence of a breakdown of general relativity at large distances and/or small accelerations.

Third, existing tests of general relativity have been restricted to *weak gravitational fields* (or arguably moderate ones in the case of the binary pulsar). Major surprises in this regime would have been surprising, since Einstein's theory goes over to Newton's in the weak-field limit, and we know that Newtonian gravity works reasonably well. But surprises are quite possible, and even likely, in the strong-field regime, where we hope to see hints about the ways in which general relativity must be modified in order to unify it with the other forces of nature.

Fourth, Einstein's theory as it stands is *incompatible with the rest of physics* (i.e. the "standard model" based on quantum field theory). The problem stems from the fact that the gravitational field carries energy and thus "attracts itself" (by contrast the electromagnetic field, for example, carries no charge). In field-theory language, the quantization of gravity requires an infinite number of renormalization parameters. It is widely believed that our present theories of gravity and/or the other interactions are only approximate "effective field theories" that will eventually be seen as limiting cases of a unified theory in which all four forces become comparable in strength at very high energies. But there is no consensus as to whether it is general relativity or particle physics–or both–that must be modified, let alone how. Experimental input may be our only guide to unification.

Gravitational experiments can be divided into two kinds: those that test fundamental principles, and those that test individual theories (including general relativity). The fundamental principles include such basic axioms as local position invariance (or LPI; the outcome of any experiment should be independent of where or when it is performed) and local Lorentz invariance (or LLI; the outcome of any experiment should be independent of the velocity of the freely-falling reference frame in which it is performed). The fundamental principle of most direct physical relevance to general relativity is the *equivalence principle*, which predicts that different test bodies should accelerate the same way in the same gravitational field, independent of their mass or internal structure, provided they are small enough not to disturb the environment or to be affected by tidal forces. The *approximate* validity of this statement has been known to some since at least the sixth century, when John Philoponous noted in a critique of Aristotle that "the ratio of the times of fall of the bodies does not depend on the ratio of their weights." It is most famously associated with Galileo at the leaning tower of Pisa. Historians of science are divided on whether that particular event actually took place, and similar ones were reported decades earlier by other people such as the Flemish engineer Simon Stevin in 1586. However, Galileo was the first to understand the significance of the measurement, and pushed it further by using a variety of different materials including gold, lead, copper and stone. He also refined the experiment by rolling his test masses down inclined tables and eventually by using pendulums.

Many people have improved on these tests since, most notably Newton and Loránd Eötvös. Newton refined Galileo's pendulum experiments, and brilliantly perceived that celestial bodies could also serve as test masses (he checked that the earth and moon, as well as Jupiter and its satellites, fall at the same rate toward the sun). This idea was reintroduced as a test of the equivalence principle by K. Nordtvedt in the 1970s, and is now applied together with laser ranging to the moon to set an upper limit on any difference in lunar and terrestrial accelerations toward the sun at less than three parts in $10^{13}$ (this is particularly significant because the earth has a nickel-iron core while the moon is largely composed of silicates). Eötvös pioneered the use of the torsion balance, enabling a six-order-of-magnitude advance in sensitivity over pendulum tests. Torsion balances are still the basis for the best equivalence-principle tests today; these limit any difference between the accelerations of different kinds of test masses in the gravitational field of the sun (or in a component of the field of the earth) to less than two parts in $10^{13}$.

**Fig. 8** The proposed Satellite Test of the Equivalence Principle (STEP)

It may be possible to reach even higher accuracy in the future through the use of laser atom interferometry to measure the rates of fall of isotopes of the same element with slightly different atomic weight. In general, however, gravitational experiments on earth are subject to inherent limitations due to factors such as seismic noise, and it is likely that further significant increases in precision will require going into space. One such proposal, the Satellite Test of the Equivalence Principle (STEP), is currently under development at Stanford University (Fig. 8). STEP is conceptually a return to Galileo's free-fall method, but one in which pairs of test masses are continuously "dropped" inside an orbiting spacecraft, allowing for a longer integration time and a periodic rather than quadratic signal. It inherits key technologies from Gravity Probe B, including drag-free control and a cryogenic readout system. STEP's design sensitivity of one part in $10^{18}$ would make it a true test, not only of the foundation of general relativity, but also of theories that attempt to unify gravity with the standard model of particle physics [53].

The "three classical tests" of general relativity were historically inaugurated by Einstein's derivation of *gravitational redshift*. In fact, this effect follows from the equivalence principle alone, so it is not a test of general relativity per se and is more properly grouped with the fundamental tests. (Some have called it the "half" in Einstein's "two and a half classical tests.") A clock in a gravitational field is, by the equivalence principle, indistinguishable from an identical one in an accelerated frame of reference. The gravitational redshift is thus equivalent to a Doppler shift between two accelerating frames. The most precise measurement of this shift to date was carried out by R. Vessot and M. Levine in 1976 (Fig. 9). Known as Gravity Probe A, their experiment compared a hydrogen maser clock on earth to an identical one in orbit at about 10,000 km and confirmed expectations based on the equivalence principle to an accuracy of 0.02%. The modern-day Global Positioning System (GPS) also functions as a de facto confirmation of gravitational redshift.

**Fig. 9** Robert Vessot and Martin Levine with the Gravity Probe A payload (1976)

GPS satellites must coordinate their time signals to about 30 ns in order to reach their specified civilian accuracy of about 10 m. This required precision in time is more than a thousand times smaller than the discrepancy between clocks on the surface and those aboard GPS satellites due to gravitational redshift, which must consequently be correct to at least 0.1% for GPS trackers to work.

The first true "classical test" of general relativity came with its successful explanation of the anomalous *perihelion shift* of the planet Mercury (the rate at which its orbit slews around the sun, as measured by its point of closest approach). This effect (along with most of the other gravitational tests) is now described in terms of a formalism invented by A. Eddington and later developed by K. Nordtvedt and C. Will into what is known as the Parametrized Post-Newtonian (PPN) framework. Here, weak, spherically-symmetric gravitational fields like that around the sun are modeled with two parameters $\gamma$ (describing the warping of space) and $\beta$ (describing the warping of time, or the nonlinearity of the theory). General relativity predicts that $\beta$ and $\gamma$ are both equal to one, and most of the experimental tests effectively place upper limits on $|\beta - 1|$ and/or $|\gamma - 1|$. Mercury's anomalous perihelion shift is proportional to $(2 + 2\gamma - \beta)/3$, which is equal to one in general relativity. Initial measurements relied on optical telescopes; modern ones are based on radar data and constrain any departure from general relativity to less than 0.3%. An important early source of systematic error came from uncertainty in solar oblateness (quadrupole moment), but this has now been well constrained from helioseismology. Perihelion shift has also been observed using radio telescopes in distant binary pulsar systems, where it is known as periastron shift.

Perihelion shift led to the rapid acceptance of general relativity among Einstein's peers but *light deflection*, the last of the three classical tests, brought him public fame. He had already found in 1911 that the equivalence principle implies some light deflection, since a beam of light sent horizontally across a room will appear to bend toward the floor if the room is accelerating upwards. In 1915, however, Einstein realized that space curvature doubles the size of the effect, and that it might be possible to detect it by observing the bending of light from background stars around the sun during a solar eclipse. Teams led by Eddington and A. Crommelin were able to confirm this prediction to an accuracy of about 30% during the eclipse of May 1919. The light deflection angle is proportional to $(1+\gamma)/2$, which is equal to one in general relativity. Constraints on $\gamma$ from optical telescopes were superseded in the 1960s by the use of linked arrays of radio telescopes (Very Long-Baseline Interferometry or VLBI) to measure the deflection around the sun of radio waves from distant quasars. By 1995 these observations had confirmed general relativity to an accuracy of 0.04%. In cosmology, light deflection (better known as gravitational lensing) is used to weigh dark matter, measure the expansion rate of the universe and even function as a cosmic "magnifying glass" to bring the faintest and most distant objects into closer view.

The space age made possible what is sometimes known as a "fourth classical test" based on the *time delay* of light signals in a gravitational field. I.I. Shapiro realized in 1964 that if general relativity was correct, then a light signal sent past the sun to a planet or spacecraft would be slowed in the sun's gravitational field by an amount proportional to the light-bending factor, $(1+\gamma)/2$, and that it would be possible to measure this effect if the signal were reflected back to earth. Typical time delays are on the order of several hundred microseconds. Passive radar reflections from Mercury and Mars were consistent with general relativity to an accuracy of about 5%. Use of the Viking Mars lander as an active radar retransmitter in 1976 confirmed Einstein's theory at the 0.1% level. The most precise of all time delay experiments to date has involved Doppler tracking of the Cassini spacecraft on its way to Saturn in 2003; this limits any deviations from general relativity to less than 0.002%–the most stringent test of Einstein's theory so far.

Radio astronomy provided a fifth test in the form of the *binary pulsar*. General relativity predicts that a non-spherically-symmetric system (such as a pair of masses in orbit around each other) will lose energy through the emission of gravitational waves. While these waves themselves have not yet been detected directly, the loss of energy has. The evidence comes from binary systems containing at least one pulsar. Pulsars are rapidly rotating neutron stars that emit regular radio pulses from their magnetic poles. These pulses can be used to reconstruct the pulsar's orbital motions. The fact that these objects are neutron stars makes them particularly valuable as experimental probes because their gravitational fields are much stronger than those of the sun (thus providing arguably "moderate-field" tests of general relativity, if not strong-field in the sense that $Gm/c^2r \sim 1$). The first binary pulsar was discovered by R.A. Hulse and J.H. Taylor in 1974. Timing measurements produce three constraints on the two unknown masses plus one more quantity; when applied to the general-relativistic energy loss formula, the results are consistent at the 0.2% level. Several other relativistic binary systems have since been discovered, including one whose orbital plane is seen almost edge-on and another in which the companion is probably a white dwarf rather than a neutron star. Most compelling is a *double pulsar* system, in which radio pulses are detected from both stars. This imposes six constraints on the two unknown masses and allows for four independent tests of general relativity. The fact that all four are mutually consistent is itself impressive confirmation of the theory. After two and a half years of observation, the most precise of these tests (time delay) verifies Einstein's theory to within 0.05%.

The perihelion-shift, light-deflection and time-delay tests firmly establish the validity of general relativity in the slow-velocity, weak-field limit within the solar system. The binary pulsar provides extra-solar confirmation of these tests and also goes some way toward extending them into the "moderate-field" regime. But there is, as yet, little accurate confirmation of Einstein's theory for strong fields such as those found near neutron-star surfaces or black-hole horizons, or over distances on the scale of the galaxy or larger. Both these difficulties may be addressed by experimental efforts aimed at the *direct* detection of gravitational waves. Most of these efforts employ interferometers to measure the difference in displacement between the lengths of two perpendicular "arms" as they are alternately stretched and compressed by the waves' passage. No gravitational waves have been detected to date. This null result does not yet impose a meaningful constraint on general relativity because of the astrophysical uncertainties inherent in predicting the strength and number of gravitational wave sources in the universe, as well as the computational challenges in modeling the characteristics of the expected signals. The strongest limits so far, from the Laser Interferometry Gravity-wave Observatory (LIGO), imply that the most frequent source events (binary neutron-star mergers) occur no more than approximately once per year per galaxy. The best theoretical estimates imply that they would not be expected more than once per $10^4 - 10^5$ years per galaxy. An upgraded version of LIGO (Advanced LIGO) is currently under construction with at least ten times the initial sensitivity.

Ground-based detectors are sensitive primarily to the high-frequency gravitational waves produced by transient phenomena (explosions, collisions, inspiraling binaries). A complementary Laser Interferometer Space Antenna (LISA) is being planned jointly by NASA and ESA; this will use a trio of spacecraft arranged in an equilateral triangle with 5 million km-long arms to look for lower-frequency waves from quasi-periodic sources, like compact objects well before coalescence and mergers between the supermassive black holes thought to lie at the centers of galaxies. LISA will rely crucially on the drag-free technology proven by Gravity Probe B. If successful, it will go a long way toward confirming the validity of general relativity, not only for strong fields but also throughout the universe.

## 5 The Geodetic and Frame-Dragging Effects

There is one other regime in which general relativity has been poorly tested to date: *spin*. Einstein's theory predicts that the spin axis of a rotating test body will precess in a gravitational field (*geodetic effect*), and that it will undergo an additional precession if the source of the gravitational field is itself rotating (*frame-dragging effect*). These phenomena might be termed the sixth and seventh tests of general relativity. Gravity Probe B is designed to confirm or disprove them directly and unambiguously for the first time. However, it is important to note that the significance of these effects goes beyond testing Einstein's theory. The question of spin is particularly important from a fundamental point of view, because it is the intrinsic spin of elementary particles that poses one of the greatest obstacles to the geometrization of standard-model fields via higher spacetime dimensions, the most promising route to unification of these fields with gravity. Thus Nobel prizewinner C.N. Yang commented in 1983 that, general relativity, "though profoundly beautiful, is likely to be amended ... whatever [the] new geometrical symmetry will be, it is likely to entangle with spin and rotation, which are related to a deep geometrical concept called torsion ... The proposed Stanford experiment [Gravity Probe B] is especially interesting since it focuses on the spin. I would not be surprised at all if it gives a result in disagreement with Einstein's theory."

The physical content of both the geodetic and frame-dragging effects can be understood in terms of analogies with electromagnetism. (Such analogies go back to Michael Faraday's experiments with "gravitational induction" beginning in 1849.) When gravitational fields are weak and velocities are low compared to $c$, then it becomes feasible to perform a "3+1 split" and decompose 4D spacetime into a scalar or 0-dimensional "time–time" component, a vector or 1-dimensional "time-space" component and a tensor or 2-dimensional "space–space" component. If one calls the scalar component a "gravito-electric potential" and the vector one a "gravito-magnetic potential," then the "gravito-electric field" $\boldsymbol{g}$ and "gravito-magnetic field" $\boldsymbol{H}$ constructed in the usual way from the divergence and curl of these potentials turn out to obey equations that are nearly identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics. Based on this analogy, the geodetic and frame-dragging effects are sometimes referred to as "gravito-electricity" and "gravito-magnetism" respectively. However, such an identification must be used with care because the distinction between gravito-electricity and gravito-magnetism depends on the frame in which it is observed, just like its counterpart in Maxwell's theory. This means that observers using different coordinate systems as, for example, one centered on the earth and another on the barycenter of the solar system, may disagree on the relative size of the effects they are discussing.

It is possible to argue that these effects have already been observed *indirectly* in the solar system, since gravito-electromagnetic fields are a necessary manifestation of Einstein's gravitational field in the low-velocity, weak-field limit, and the validity of general relativity is now routinely assumed in, for instance, updating the ephemeris of planetary positions. In this sense, it would be surprising if an experiment like Gravity Probe B, which is designed to observe gravito-electromagnetic effects *directly*, did not see them. Such a result would suggest that general relativity needs to be extended or modified in some way such that terms involved in the geodetic and/or frame-dragging effects are strongly affected while leaving predictions for other post-Newtonian effects unchanged. Nevertheless, surprises do occur in science, and a surprise here would have major implications for unification of gravity with the rest of physics. On such fundamental questions, history has shown that there is no substitute for the direct test.

Symmetry considerations dictate that the earth's gravito-electric field must be radial and its gravito-magnetic one dipolar (Fig. 10). From these facts one can immediately write down the precessions due to the geodetic effect ($\boldsymbol{\Omega}_{\mathrm{g}}$) and

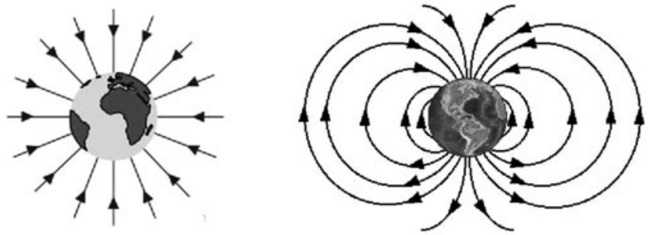

**Fig. 10** The earth's radial gravito-electric field (*left*) and dipolar gravito-magnetic field (*right*)

frame-dragging effect ($\boldsymbol{\Omega}_{\rm fd}$) by referring to standard formulae governing the motion of a test charge in an external electromagnetic field, and replacing the electric and magnetic fields by $\boldsymbol{g}$ and $\boldsymbol{H}$ respectively. The result is:

$$\boldsymbol{\Omega}_{\rm GR} = \boldsymbol{\Omega}_{\rm g} + \boldsymbol{\Omega}_{\rm fd} = \frac{3GM}{2c^2r^3}(\boldsymbol{r} \times \boldsymbol{v}) + \frac{GI}{c^2r^3}\left[\frac{3\boldsymbol{r}}{r^2}(\boldsymbol{S}\cdot\boldsymbol{r}) - \boldsymbol{S}\right], \qquad (3)$$

where $M$, $I$ and $\boldsymbol{S}$ refer to the mass, moment of inertia and angular momentum of the central body and $\boldsymbol{r}$ and $\boldsymbol{v}$ are the radial position and instantaneous velocity of the test body. Equation (3) is sometimes referred to as the *Schiff formula* after Leonard I. Schiff, who derived it in 1959.

The geodetic or $\boldsymbol{r} \times \boldsymbol{v}$ term in (3) arises from the way that angular momentum is transported through a gravitational field. Einstein's Dutch friend and colleague Willem de Sitter (1872–1934; Fig. 11) began to study this problem in 1916 when general relativity was less than a year old. He found that the orbital angular momentum of the earth-moon system precesses in the field of the sun, a special case now referred to as the de Sitter or "solar geodetic" effect (although "heliodetic" might be more descriptive). De Sitter's calculation was extended to the spin angular momentum of rotating test bodies by two of his countrymen: in 1918 by the mathematician Jan Schouten (1883–1971) and in 1920 by the physicist and musician Adriaan Fokker (1887–1972). Eddington brought these results to the attention of the wider community in *The Mathematical Theory of Relativity* (1923), writing that "If the earth's rotation could be accurately measured ... by gyrostatic experiments, the result would differ from the rotation relative to the fixed stars." This was the germ of the idea that would eventually grow into Gravity Probe B.

In the framework of the gravito-electromagnetic analogy, the geodetic effect can be seen partly as a spin-orbit interaction between the spin of the gyroscope and the "mass current" of the rotating earth. This is the analog of Thomas precession in electromagnetism, where the electron experiences an induced magnetic field due to the apparent motion of the nucleus around it (in its rest frame). In the gravitomagnetic case, the gyroscope "feels" the massive earth orbiting around it (in its rest frame) and experiences an induced *gravito*-magnetic torque, causing its spin vector to precess. This spin-orbit interaction accounts for one third of the total geodetic precession; the other two thirds arise due to space curvature alone and cannot be easily interpreted

**Fig. 11** Discoverers of the geodetic effect in general relativity: Willem de Sitter (*left*), Jan Schouten (*center*) and Adriaan Fokker (*right*)

gravito-electromagnetically. They can however be easily understood geometrically (Fig. 12). The gyroscope's spin vector remains always perpendicular to its plane of motion (arrows), and in flat space its direction remains constant as the gyroscope completes an orbit (left). If, however, space is folded into a cone to simulate the effect of curvature, then part of the area of the circle (shaded) must be removed and the gyroscope's spin vector no longer lines up with itself after one making one complete circuit (right). The angle between the spin vectors "before" and "after" produces the other two thirds of the geodetic effect. In the case of Gravity Probe B this is sometimes referred to as the phenomenon of the "missing inch" because space curvature shortens the circumference of the spacecraft's orbital path around the earth by 1.1 in. In polar orbit at an altitude of 642 km the total geodetic effect (comprising both the spin-orbit and space curvature effects) causes a gyroscope's spin axis to precess in the north-south direction by 6,606 milliarcseconds over the course of a year–an angle so small that it is comparable to the average angular size of the planet Mercury as seen from earth.

Experimental limits on geodetic precession place new constraints on a broad class of alternatives to Einstein's theory of gravity known as "metric theories" (loosely speaking, theories that differ from Einstein's but still respect the equivalence principle). These are characterized by the PPN parameters $\beta$ and $\gamma$, both equal to one in general relativity [55]. The geodetic effect is proportional to $(1 + 2\gamma)/3$, so an experimental detection translates directly into a constraint on $\gamma$. It also probes other kinds of "generalizations of general relativity" such as those involving extra spacetime dimensions [35], scalar fields [56], torsion [57, 58] and violations of Lorentz invariance, the conceptual foundation of special relativity [59, 60].

The frame-dragging or $\boldsymbol{S}$-dependent term in (3) is smaller in magnitude than the geodetic one, but reveals more clearly the Machian aspect of Einstein's theory. In fact, it is curious that Einstein did not discover this effect himself, given that he had explicitly looked for dragging phenomena in his earlier attempts at gravitational field theories, and that he still attached enough importance to Mach's principle to refer to it as a pillar of general relativity in 1918. For whatever reason, frame-dragging within general relativity was first discussed that same year by Austrian physicists Hans Thirring (Fig. 13; 1888–1976) and Josef Lense (1890–1985); it is

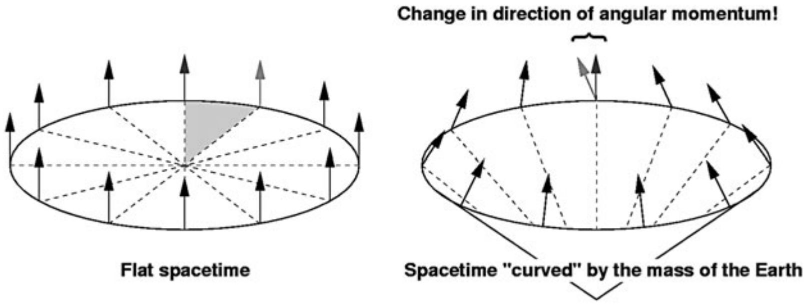


**Fig. 12** Geodetic precession and the "missing inch"

often referred to as the Lense-Thirring effect. Thirring originally approached this problem as an experimentalist; he hoped to look for Mach-type dragging effects inside a massive rotating cylinder. Unable to raise the necessary financing, he reluctantly settled down to solve the problem theoretically instead [61]. It is his second calculation (with Lense) involving the field outside a slowly rotating solid sphere that forms the basis for modern gyroscopic tests. But both his results are "Machian" in the sense that the inertial reference frame of the test particle is influenced by the motion of the larger mass (the cylinder or sphere). This is completely unlike Newtonian dynamics, where local inertia arises entirely due to motion with respect to "absolute space" and is unaffected by the distribution of matter.

In terms of the gravito-electromagnetic analogy, frame-dragging is a manifestation of the *spin–spin* interaction between the test body and central mass. It is analogous to the interaction of a magnetic dipole $\boldsymbol{\mu}$ with a magnetic field $\boldsymbol{B}$ (the basis of nuclear Magnetic Resonance Imaging or MRI). Just as a torque $\boldsymbol{\mu} \times \boldsymbol{B}$ acts in the magnetic case, so a test body with spin $\boldsymbol{s}$ experiences a torque proportional to $\boldsymbol{s} \times \boldsymbol{H}$ in the gravitational case. In the case of Gravity Probe B, this torque causes the gyroscope spin axes to precess in the east-west direction by 39 milliarcseconds per year–an angle so tiny that it is equivalent to the average angular width of the dwarf planet *Pluto* as seen from earth.

The orbital plane of an artificial satellite is also a kind of "gyroscope" whose nodes (the points where it intersects a reference plane) will exhibit a similar frame-dragging precession (the de Sitter effect). Such an effect has been reported in the case of the earth-orbiting Laser Geodynamic Satellites (LAGEOS and LAGEOS II) by Ignazio Ciufolini and colleagues using laser ranging [62, 63]. This method of looking for frame-dragging is elegant and complementary to the more direct gyroscopic test. It is not definitive on its own because the general-relativistic effect (31 milliarcseconds per year at the LAGEOS altitude of 59,000 km) is swamped by Newtonian contributions that are as much as a billion times larger. To model or otherwise remove these terms necessarily involves systematic uncertainties whose magnitude is still a subject of debate [64–66].

In principle, frame-dragging imposes another new constraint on alternative metric theories of gravity. Lense-Thirring precession is proportional to the combination

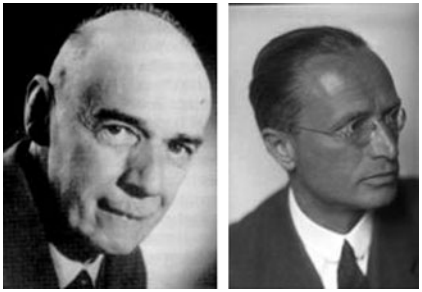

**Fig. 13** Discoverers of the frame-dragging effect in general relativity: Josef Lense (*left*) and Hans Thirring (*right*)

of PPN parameters $(\gamma+1+\alpha_1/4)/2$ where $\gamma$ describes the warping of space and $\alpha_1$ is a "preferred-frame" parameter that allows for a possible dependence on motion relative to the rest frame of the universe, taking the value zero in general relativity [55]. In practice, frame-dragging in the solar system is so weak that the experimental bounds it places on these parameters are not likely to be competitive with those from other tests. Seen purely as a testbed for distinguishing between alternative theories of gravity, therefore, frame-dragging has sometimes been dismissed as being of little practical interest.

That way of thinking has largely disappeared with the realization that frame-dragging takes on its true importance in the strong-field and cosmological regimes. Astrophysicists now invoke gravitomagnetism as the engine and alignment mechanism for the vast jets of gas and magnetic field ejected from quasars and galactic nuclei like the radio source NGC 6251 (Fig. 14, left). These jets are generated by compact objects at the centers of galactic nuclei that are almost certainly supermassive black holes (right). The megaparsec length scale of the jets implies that their direction is held constant over time scales as long as tens of millions of years. This can only be accomplished by the gyroscopic spin of the black hole, and the only way the direction of that spin can be communicated to the jet is via the black hole's gravitomagnetic field $\boldsymbol{H}$ [54]. The field causes the accretion disk to precess around the black hole, and that precession combines with the disk's viscosity to drive the inner region of the disk into the hole's equatorial plane, gradually forcing the jets to align with the north and south poles of the black hole. This phenomenon, known as the Bardeen-Petterson effect, is widely believed to be the physical mechanism responsible for jet alignment.

Gravitomagnetism is also thought to lie behind the generation of the astounding energy contained in these jets in the first place. The event horizon of the black hole can act like a gigantic "battery" where the gravitomagnetic potential of the black hole interacts with the tangential component of the ordinary magnetic field $\boldsymbol{B}$ to produce a drop in electric potential [54]. This phenomenon, known as the Blandford-Znajek mechanism, effectively converts the immense gravitomagnetic, rotational energy of the supermassive black hole into an outgoing stream of ultra-relativistic charged particles. Gravity Probe B has thus become a test of the mechanism that powers the most violent explosions in the universe.


**Fig. 14** Megaparsec-scale jet associated with the strong radio source NGC 6251 (*left*); such jets are now thought to be aligned and powered by the gravitomagnetic fields of rotating supermassive black holes (*right*)

It is on cosmological scales, however, that frame-dragging may take on its deepest significance, as part of the explanation for the origin of inertia. Consider the earth's equatorial bulge. Textbooks teach us that this phenomenon is due to rotation with respect to the "inertial frame" in which the universe as a whole happens to be at rest (Fig. 15, left). But what if it were the *earth* that stood still, and the rest of the universe that rotated (right)? Would the equator still bulge? Newton would have said "No." For him, inertial frames were tied ineluctably to absolute space which "in its own nature, without relation to anything external, remains always similar and immoveable." We know that the concept of absolute space (time) is retained in general relativity, so we might have expected that the same answer would carry over to Einstein's theory as well. However, it does not. As demonstrated by Thirring in his original calculation of 1918, and amplified by many others since, general-relativistic frame-dragging goes over to "perfect dragging" *when the dimensions of the large mass become cosmological*. That is, if the entire universe were to rotate, it would drag the inertial frame of the earth around with it. On this basis, Einstein would have had to answer "Yes" to the question posed above. In this respect general relativity is indeed more relativistic than its predecessors, as Mach would have wished. Early calculations were flawed in many ways, but the phenomenon of perfect dragging has persisted through a series of increasingly sophisticated treatments, notably those of Sciama [8], Brill and Cohen [67,68], Lindblom and Brill [69], Pfister and Braun [70] and Klein [71,72]. Pfister sums up the situation as follows [61]: "Although Einstein's

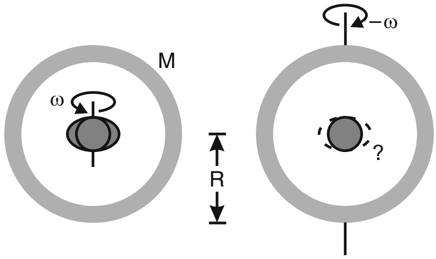


**Fig. 15** Would the earth still bulge, if it were standing still and the rest of the universe were rotating around *it?*

theory of gravity does not, despite its name 'general relativity,' yet fulfil Mach's postulate of a description of nature with only relative concepts, it is quite successful in providing an intimate connection between inertial properties and matter, at least in a class of not too unrealistic models for our universe." The apparently *instantaneous* nature of the connection is particularly mysterious. Much remains to be learned. What is clear, however, is that direct detection of frame-dragging by Gravity Probe B will do much to give us confidence in what has been a largely theoretical enterprise to date; namely, to understand how a relational explanation for inertia may be possible within a theory of absolute spacetime.

## 6 Gravity Probe B

The English physicist P.M.S. Blackett reportedly considered the idea of looking for the de Sitter effect with a laboratory gyroscope as early as the 1930s [73]. The smallness of the signal, however, put such an experiment far out of reach until after post-World War II improvements in gyroscope technology and the dawning of the space age. To measure a yearly precession of order 10 milliarcseconds to 1% accuracy requires a gyroscope with drift rate less than $10^{-18}$ rad/s. On earth, where (for instance) density inhomogeneities contribute to this drift rate with the full force of the earth's gravitational acceleration $a \sim g$, the gyro rotor would have to be homogeneous to a part in $\sim 10^{17}$–a hopelessly unattainable number. A similar argument holds for rotor asphericity. The only way around such fundamental limitations is to go into space, where unwanted accelerations can be suppressed–with a great deal of work–so that $a \sim 10^{-11} g$. The rotor need then be homogeneous, for example, to "only" one part in $\sim 10^6$, a level that can be achieved, with great effort, using the best materials on earth [73].

Considerations of this kind led two American physicists to take a new look at gyroscopic tests of general relativity independently within months of each other in late 1959 and early 1960. George E. Pugh (b. 1928; Fig. 16) was spurred by a talk given by the Turkish-American theoretical physicist Huseyin Yilmaz on the possible use of an artificial satellite to distinguish his theory of gravity from Einstein's. He noted that such an experiment "would be, in the most literal sense a direct measurement of space itself" [74]. Leonard. I. Schiff (1915–1971) was inspired at least in

**Fig. 16** Leonard Schiff c. 1970 (*top left*), George Pugh in 2007 (*bottom left*) and Dan Debra, Bill Fairbank, Francis Everitt and Bob Cannon with a model of Gravity Probe B in 1980 (*right*)

part by a magazine advertisement for a new "Cryogenic Gyro ... with the possibility of exceptionally low drift rates" [73]. Schiff had a longstanding interest in both general relativity and Mach's principle, and went so far as to refer to his proposal as "an experimental test of Mach's principle" [75]. He was joined by low-temperature experimentalist Bill Fairbank and guidance and control specialist Bob Cannon, and together the three men set Gravity Probe B on the path to reality. Under its original name (the Stanford Relativity Gyroscope Experiment) the project received its first NASA funding in 1964.

Pugh's paper attracted less notice at the time but is now recognized as the birth of the concept of *drag-free motion*. This is a critical element of the Gravity Probe B mission, whereby any one of the gyroscopes can be isolated from the rest of the experiment and protected from non-gravitational forces (such as those caused by solar radiation pressure and atmospheric drag); the rest of the spacecraft is then made to "chase after" the reference gyro by means of helium boiloff vented through a revolutionary porous plug and specially designed thrusters. Gravity Probe B's demonstration that cross-track accelerations can be suppressed in this way to less than $10^{-11}g$ paves the way for the development of future gravitational experiments such as the Satellite Test of the Equivalence Principle (STEP) and the Laser Interferometer Space Antenna (LISA). The porous plug has already proved vital to other cryogenic NASA missions including COBE, IRAS, WMAP and Spitzer.

The experimental concept is illustrated in Fig. 17. In principle it is simplicity itself: a gyroscope, a readout mechanism to monitor the spin axes, and a telescope to compare these axes with the line of sight to a distant guide star. In practice, Gravity Probe B evolved into one of the most complex experiments ever flown, requiring at least a dozen new technologies that did not exist when it was conceived. How, for instance, is one to locate the spin axis of a perfectly spherical, perfectly homogeneous gyroscope, suspended in vacuum (Fig. 18)? This is the readout problem; another, closely related challenge is how to spin up such a gyroscope in the first place. Various possibilities were considered in the early days, until 1962 when

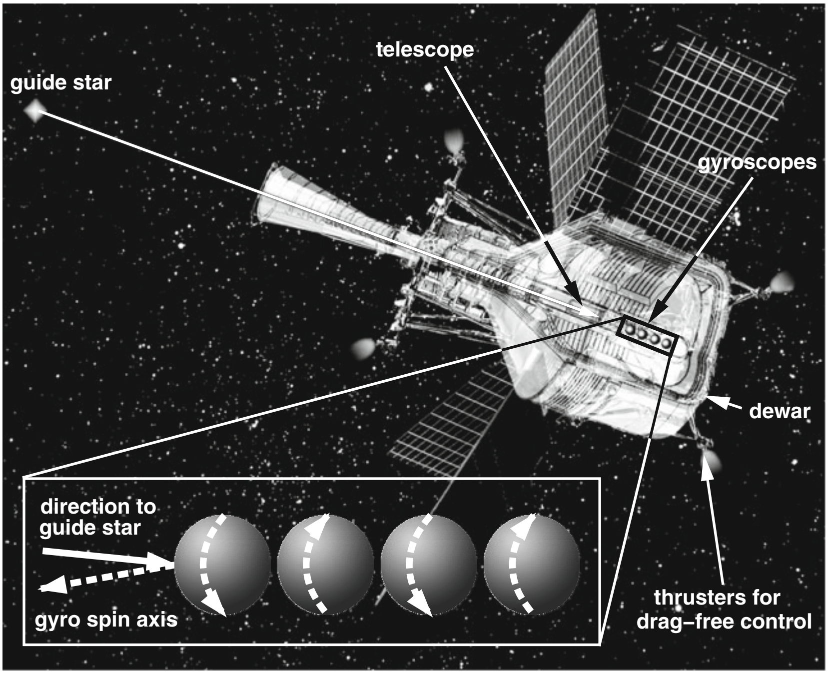


**Fig. 17** The Gravity Probe B concept, described by William Fairbank, one of the project founders, as "just a star, a telescope and a spinning sphere"

C.W. Francis Everitt (now in charge of the experiment) hit on the idea of exploiting what had until then been a small but annoying source of unwanted torque in magnetically levitated gyroscopes. Spinning superconductors develop a magnetic moment, known as the *London moment*, which is aligned with the spin axis and proportional to the spin rate. If the rotors were levitated electrically instead of magnetically, this tiny effect could be used to monitor their spin axes. (Measuring it would require magnetic shielding orders of magnitude beyond anything available in 1962, another story in itself.) Thus was born the London moment readout, which in its modern incarnation uses niobium-coated quartz spheres as rotors and SQUIDs (Superconducting QUantum Interference Devices) as magnetometers. So sensitive are these devices that they register a change in spin-axis direction of 1 milliarcsecond in just five hours of integration time.

How can one meaningfully compare the spin-axis direction (from the SQUID, in volts) with the direction to the guide star (from an onboard telescope, in radians)? The answer is to exploit nature's own calibration in the form of *stellar aberration*. This phenomenon, an apparent back-and-forth motion of the guide star position due to the orbit of the earth around the sun, is entirely Newtonian and inserts "wiggles" into the data whose period and amplitude are exquisitely well known. (Such is the precision of the experiment that this calibration requires terms of second, as well as

**Fig. 18** Gravity Probe B gyroscope rotor and housing. Note suspension electrodes (circular patterns) and gas spin-up channel (groove)

first order in the earth's speed $v/c$.) What about the fact that the guide star *itself* has an unknown proper motion large enough to obscure the predicted relativity signal? This apparent liability is turned into an advantage by designing the experiment in classic "double-blind" fashion: a separate team of radio astronomers uses VLBI to monitor the movements of the guide star relative to even more distant quasars. Only at the conclusion of the experiment are the two sets of data to be compared; this helps to prevent the physicists from finding what they expect to see.

Necessity in the form of a $10^{-18}$ rad/s precession rate was the mother of many more marvels. Among these are the roundest man-made objects in the world and a suspension system capable of keeping them within microns of their housings, at spin rates averaging 4,000 rpm, over a dynamic range of eight or more orders of magnitude in force. A beam splitter and image divider assembly was created to increase the resolution of the onboard telescope (inherently limited by the size of the spacecraft) by three orders of magnitude over existing star trackers. A novel optical bonding technique had to be devised to fasten the telescope (sculpted out of a single lump of quartz) to the quartz block containing the science instrument. Expandable nested lead shields were employed to reduce the strength of the magnetic field inside the dewar to less than one-millionth that of the Earth, the lowest level ever achieved in space. New techniques were invented to spin up the gyros, reduce vacuum pressure and remove charge buildup on the rotors. Many of these innovations have led to engineering and commercial spinoffs.[7]

Gravity Probe B was launched from Vandenberg Air Force Base in California on 20 April 2004 (Fig. 19). Once in orbit, it underwent an initial orbit checkout phase, during which the attitude and control system was tuned and the gyroscopes suspended, spun up, calibrated and aligned with the guide star. These tasks required 129 days. The science or data-collecting phase of the mission lasted from 27 August 2004 until 14 August 2005, or 353 days, just under the original goal of one full year. The mission concluded with a final post-flight calibration phase, which continued until 29 September 2005, when there was no longer enough liquid helium in the dewar to maintain the experiment at cryogenic temperatures.

[7] For more details, see the Gravity Probe B website at http://einstein.stanford.edu.

**Fig. 19** Launch of Gravity Probe B at 09:57:24 PDT, April 20, 2004

Figure 20 shows approximately 140 days of science data from one of the gyroscopes (points) superimposed on the predictions of general relativity (lines). North-south (geodetic) precession is plotted in the upper panel, while east-west (frame-dragging) precession is plotted in the lower panel. These plots give us *our first direct look at the warping and twisting of spacetime around the earth.* If Newton were correct, the data would fall on horizontal lines.

As might be expected in an experiment that pushes gyroscope performance six orders of magnitude beyond existing limits, unexpected complications have cropped up in the data analysis. First, it became apparent during the science phase of the mission that there were variations in the polhode rate of the gyros. (Polhode motion had been expected, but its period had not been expected to change appreciably over the mission lifetime, given characteristic rotor spin-down periods on the order of 10,000 years). It is critical to understand and model these polhode variations in order to match the data from successive orbits and thereby attain integration times long enough to realize the full precision of the SQUID readout system. Second, two larger-than-expected forms of Newtonian torque, known as the "misalignment" and "roll-polhode resonance" torques, were discovered during post-flight calibration. Misalignment torques were proportional to the angle between the gyroscope spin axis and the spacecraft roll axis, while resonance torques acted on individual gyroscopes during times when there was a high-order resonance between the slowly changing polhode period and the satellite roll period. All three phenomena have been traced to larger-than-anticipated electrostatic patch effects. In essence, while both the gyro rotors and housings achieved almost perfect *mechanical* sphericity, they were not quite spherical *electrically*. The anomalous torques are due to

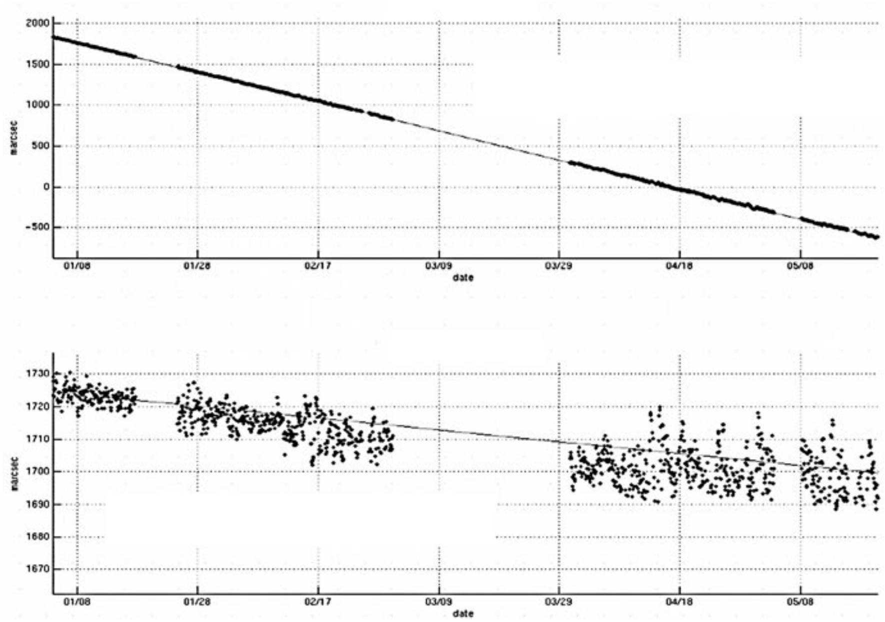


**Fig. 20** Preliminary results for the precession of one of the Gravity Probe B gyroscopes in the north-south or geodetic direction (*top*) and the east–west or frame-dragging direction (*bottom*)

interactions between patches on the gyro rotors and housings, and the time-varying polhode periods are caused by the fact that these interactions extract energy from the spinning rotors.

Fortunately, Gravity Probe B was designed to take various kinds of "superfluous" data, and these are now proving their worth. In particular, real-time snapshots of trapped flux on the rotors have enabled the data analysis team to reconstruct the polhode phase of each of the gyros to within $\sim 1°$ over the entire mission. Spin speeds are known to $\sim 1$ nHz, and spin-down rates to $\sim 1$ pHz/s. With this data, in combination with a complete physical understanding of all three (fully Newtonian) effects, it has been possible to develop a more comprehensive method of data analysis that is expected to lead to final accuracies close to those originally envisioned for the experiment. Table 1 summarizes interim results from all four gyroscopes as of December 2008 [76]. These numbers are preliminary and do not include all sources of systematic error or model sensitivity analysis. Nevertheless it is possible at this stage to state that geodetic precession has been directly observed at better than 1%, and that frame-dragging has been directly observed with an accuracy of about 15%.

Final results from Gravity Probe B are to be announced in 2010.

**Table 1** Preliminary Gravity Probe B results (milliarcsec/yr) [76]

| | | Solar geodetic | Guide star proper motion | Net predicted ($\Omega_{GR}$) | Observed ($\Omega_{obsd}$) |
|---|---|---|---|---|---|
| North–south: | −6606 | +7 | +28 ± 1 | −6571 ± 1 | −6550 ± 14 |
| East–west: | −39 | −16 | −20 ± 1 | −75 ± 1 | −69 ± 6 |

## 7 Summary

The detection of geodetic precession and frame-dragging by Gravity Probe B can be seen as the culmination of a debate that stretches back to Greek antiquity. That debate was originally philosophical: do space and time exist absolutely, or only in relation to matter? As natural philosophy evolved into natural science, it began to take on a physical character. The absolute picture, advocated most forcefully by Newton, was physically simpler but carried with it uncomfortable metaphysical baggage (inertia as resistance to motion with respect to "absolute space," which itself could neither be observed nor acted upon in any way). The relational view, most strongly associated with Mach, was philosophically more elegant but troublingly vague in the physical sense (what kind of relation, exactly, gives rise to inertia?) Attempts were already made to distinguish between the two points of view by experimentalists such as Föppl before the time of Einstein and Minkowski.

With the advent of general relativity, it became possible to frame the debate in precise physical terms. It turned out that Minkowski's spacetime, as shaped and animated in the presence of matter according to Einstein's gravitational field equations, took neither side in the debate–or rather, took them both. The spacetime of general relativity exists absolutely and behaves relationally, as exemplified by the geodetic and (especially) frame-dragging effects.

Ninety years of experiment have solidified the case for Einstein's theory. However, most of the evidence so far is limited to the solar system where fields are weak and velocities low. Gravitational-wave astronomy has the potential to improve the situation, as do experiments that challenge the foundations (as opposed to predictions) of general relativity, like tests of the equivalence principle.

The geodetic and frame-dragging effects test Einstein's theory in another direction by focusing on the *spin* of the central mass and test body, with important implications for astrophysics, cosmology and the origin of inertia. Such is their subtlety, however, that detecting them with confidence has required 40 years of scientific and engineering ingenuity and perseverance. That story is not quite finished, and may yet fulfil the original aim of the Gravity Probe B mission: to provide the "most rigorously validated of all tests of Einstein's theory" [76]. Preliminary data are consistent with general relativity. These results tighten constraints on alternative theories of gravity, improve confidence in astrophysical models of the jets and accretion disks associated with supermassive black holes, and suggest that we may be close to understanding why our local compass of inertia is aligned with the rest frame of the distant galaxies.

They also settle an old debate in metaphysics. It would be hard to imagine a more direct demonstration that spacetime acts on matter than the geodetic effect (warped spacetime twists a spinning gyroscope), or a more convincing proof that matter acts back on spacetime than the frame-dragging effect (the spinning earth pulls spacetime around with it). In that sense Gravity Probe B shows how a physics experiment–when pushed to the furthest possible extremes of near-zero temperature, pressure, electric charge, magnetic field and acceleration–can also become a test of philosophy.

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